Autonomous Area Search in the Framework of Possibility Theory

Zhijin Chen
School of Engineering
RMIT University
Melbourne, VIC, Australia
s3762807@student.rmit.edu.au

Branko Ristic
School of Engineering
RMIT University
Melbourne, VIC, Australia
branko.ristic@rmit.edu.au

Du Yong Kim
School of Engineering
RMIT University
Melbourne, VIC, Australia
duyong.kim@rmit.edu.au

***Abstract*—The paper formulates the solution to area search for targets in the framework of possibility theory. The rationale is that the required measurement model parameters, such as the probability of detection and/or the probability of false alarm, are rarely known as precise values. Possibility theory was developed for quantitative modelling of and reasoning with *epistemic uncertainty*. It provides an elegant Bayesian like solution to target area search. A reward function is proposed as an uncertainty measure which takes into account the epistemic uncertainty. The robustness of the proposed search algorithm is demonstrated by numerical results.**

I. Introduction

Search problems arise in many domains of life, such as, for example, rescue operations, security operations (e.g. search for toxic or radioactive emissions), understanding animal behaviour, and military operations (anti-submarine warfare) [1]. Search is a repetitive cycle of sensing, estimation, decision making and motion action [2]. Recently, search techniques have been applied in field robotics [3], [4], for the purpose of carrying out dirty and dangerous missions. A formal search theory has roots in the works by Koopman [5], and has since expanded and extended to different problems and application. It can be categorised into a static versus a moving target search, a reactive versus a non-reactive target search, a single versus multiple target search, a cooperative versus a non-cooperative target search, etc [1].

In this paper we focus on an area search for an unknown number of static targets using a realistic sensor on a single searching platform, conceptually similar to the problems discussed in [6]–[8]. The searching agent is a drone, equipped with a sensor capable of detecting targets on the ground with a certain probability of detection (as a function of range) as well with some false alarm probability. Recently, unmanned aerial vehicles (UAVs) have been deployed in typical surveillance and search missions [9], [10].

The dominant theoretical framework for search is based on probability theory [2], [11]–[13], where Bayesian inference is used to update sequentially the posterior probability distribution of target locations, as the new measurements are collected over time [8], [14]. Sensor motion control is typically formulated as a partially observed Markov decision process (POMDP) [15]. The information state in the POMDP formulation is represented by the posterior probability distribution of targets. The set of sensor motion controls (actions), which determine where the searching agent should move next, can be made by a single or multiple steps ahead. The reward function in POMDP maps the set of admissible actions to a set of positive real numbers (rewards) and is typically formulated as a measure of information gain (e.g. the reduction of entropy, Fisher information gain) [16].

The Bayesian inference framework requires precise probabilistic models. In the target search context, we need a model of sensing, which incorporates the uncertainty with regards to the probability of true and false detection as well as the statistics of target (positional) measurement errors. This uncertainty in the Bayesian framework is expressed by probability functions - in particular, the probability of detection, the probability of false alarm, and the probability density function (PDF) of a positional measurement, given the true target location. The key limitation of the Bayesian approach, however, is that these probabilistic models must be known *precisely*. Unfortunately, in many practical situations it is difficult or even impossible to postulate the precise probabilistic models. Consider for example the probability of detection. It typically depends on the (unknown) size and reflective characteristics of the target, and hence at best can be specified as a probability or confidence interval (rather than a precise probability value), for a given distance to the target. Thus we need to deal with epistemic uncertainty, which incorporates both randomness and partial ignorance.

In order to deal with epistemic uncertainty, an alternative mathematical framework for inference is required. These theories involve *non-additive* probabilities [17] for the representation and processing of uncertain information. They include, for example, possibility theory [18], Dempster-Shafer theory [19] and imprecise probability theory [20]. Because the last two theories are fairly complicated, and at present applicable only to discrete state spaces, we focus on possibility theory [21], [22]. Recent research in nonlinear filtering and target tracking [23]–[27] have demonstrated that possibility theory provides an effective tool for uncertain knowledge representation and reasoning.

The contribution of this paper is a novel autonomous search algorithm formulated in the framework of possibility theory. The main feature of this approach is that the current knowledge (information) about the number and location of targets is represented by the difference between the posterior possibility of target presence and target absence. In addition, a reward function for the searcher motion control is proposed as an epistemic uncertainty measure. Evaluation of the proposed search algorithm considers the scenarios with a large number of targets and two cases of the probability of detection P_{D} as a function of range: (i) the case when P_{D} is known precisely; (ii) the case when P_{D} is known only as an interval value.

The paper is organised as follows. Sec. II introduces formally the autonomous search problem. Sec. III reviews the standard probabilistic formulation of autonomous search and presents the fundamentals of possibilistic estimation framework. Sec. IV formulates the new possibilistic solution to autonomous search. Numerical results with comparison are presented in Sec. V, while the conclusions are drawn in Sec. VI.

II. Problem formulation

Consider a designated search (surveillance) area $\mathcal{S}$. A surveillance drone flies at a fixed altitude and searches for ground-based static targets, as in [8]. The number and locations of targets is unknown. The problem is a sequential target localisation and two-dimensional path-planning of the search trajectory.

As in [7], [8], the search area $\mathcal{S}$ is discretized into $n_c \gg 1$ cells of equal size. The presence or absence of a target in the nth cell at a discrete-time k can be modelled by a Bernoulli random variable (r.v.) $X_{k,n} \in \{0,1\}$, where by convention $X_{k,n} = 1$ denotes that a target is present, (i.e. 0 denotes target absence) and $n = 1,\dots,n_c$.

Suppose the search agent (the surveillance drone) is equipped with a sensor (e.g. a radar), which illuminates a region $\mathcal{L}_k \subset \mathcal{S}$ at the time k and collects a set of detections (or measurements) $\mathbf{Z}_k$. Each detection reports the Cartesian coordinates of a possible target. The sensing process, however, is uncertain in two ways: (1) the reported target coordinates are affected by measurement noise; (2) the measurement set $\mathbf{Z}_k$ may include false detections and may miss some of the true target detections. The probability of true target detection is a (monotonically decreasing) function of range and is specified as an interval value for a given range.

The objective is to detect and localise as many targets as possible in the shortest possible time.

III. Background

A. Probabilistic search

Autonomous cognitive search in the Bayesian probabilistic framework is typically information driven. The information state at time k is represented by the posterior probability of target *presence* in each cell of the discretised search area. This posterior probability at time k is denoted by $P_{k,n} = Pr\{X_{k,n} = 1|\mathbf{Z}_{1:k}\}$, where $n = 1,\dots,n_c$ is the cell index and $\mathbf{Z}_{1:k} := \mathbf{Z}_1,\cdots,\mathbf{Z}_k$ is the sequence of measurement sets up to the current time k. The posterior probability of target *absence* is then simply $\bar{P}_{k,n} = Pr\{X_{k,n} = 0|\mathbf{Z}_{1:k}\} = 1 - P_{k,n}$, and therefore is unnecessary to compute.

The threat map is defined as the array $\mathbb{P}_k = [P_{k,n}]$, where $n = 1,\dots,n_c$. It is updated using the Bayes' rule. Initially, at time $k = 0$, $P_{0,n} = \frac{1}{2}$, for all $n = 1,\dots,n_c$, thus expressing the initial ignorance. As time progresses the threat map is updated using new measurement sets and its information content is increasing. The information content of the threat map is measured by its entropy, defined as

$$\mathcal{H}_k = \\ -\frac{1}{n_c}\sum_{n=1}^{n_c}\left[P_{k,n}\log_2 \mathrm{P_{n,k}} + (1-\mathrm{P_{n,k}})\log_2(1-\mathrm{P_{n,k}})\right]. \tag{1}$$

Entropy of the threat map at time $k = 0$ equals $\mathcal{H}_0 = 1$, and decreases over time.

Given $\mathbb{P}_{k-1}$, if none of the detections in $\mathbf{Z}_k$ falls into the nth cell, the probability $P_{n,k}$ is updated as

$$P_{k,n} = \frac{(1-D_{k,n})P_{k-1,n}}{(1-D_{k,n})P_{k-1,n} + (1-F_{k,n})(1-P_{k-1,n})} \tag{2}$$

where $D_{k,n}$ is the probability of detection and $F_{k,n}$ is the probability of false-alarm, in the nth cell of search area $\mathcal{S}$ at time k.

If $\mathbf{Z}_k$ contains a detection in the nth cell, then the update equation is given by

$$P_{k,n} = \frac{D_{k,n}P_{k-1,n}}{D_{k,n}P_{k-1,n} + F_{k,n}(1-P_{k-1,n})} \tag{3}$$

Eqs. (2) and (3) are expressions of the Bayes rule.

After collecting the measurement set $\mathbf{Z}_{k-1}$, the searching agent must decide on its subsequent action, that is, where to move (and sense) next. Suppose the set of possible actions (for movement) is $\mathcal{A}_k$. This set can be formed by considering one or more motion steps ahead (in the future). The reward function associated with every action $\alpha \in \mathcal{A}_k$ is typically defined as the reduction in entropy of the threat map [8], that is:

$$\mathcal{R}_k(\alpha) = \mathcal{H}_{k-1} - \mathcal{E}\{\mathcal{H}_k(\alpha)\} \tag{4}$$

Note that the expectation operator $\mathcal{E}$ with respect to $p(\mathbf{Z}_k(\alpha)|\mathbf{Z}_{1:k-1})$ features in (4). Finally, the searching agent chooses the action which maximises the reward, i.e.:

$$\alpha_k^* = \arg\max_{\alpha\in\mathcal{A}_k} \mathcal{R}_k(\alpha). \tag{5}$$

B. The possibilistic estimation framework

Possibility theory is developed for quantitative modelling of *epistemic uncertainty*. The concept of *uncertain variable* in possibility theory, plays the same role as the random variable in probability theory. The main difference is that the quantity of interest is not random, but simply unknown, and our aim is to infer its true value, out of a set of possible values. The theoretical basis of this approach can be found in [28], [29]. Briefly, uncertain variable is a function $X : \Omega \to \mathcal{X}$, where Ω is the sample space and $\mathcal{X}$ is the state space (the space where the quantity of interest lives). Our current knowledge about X can be encoded in a function $\pi_X : \mathcal{X} \to [0,1]$, such that $\pi_X(x)$ is the possibility (credibility) for the event $X = x$. Function π_X is not a density function, it is referred to as a possibility function, being the primitive object of possibility theory [30]. It can be viewed as a membership function determining the fuzzy restrictions of minimal specificity (in the sense that any hypothesis not known to be impossible cannot be ruled out) about x [18]. Normalization of π_X is $\sup_{x\in\mathcal{X}} \pi_X(x) = 1$, if $\mathcal{X}$ is uncountable, and $\max_{x\in\mathcal{X}} \pi_X(x) = 1$, if $\mathcal{X}$ is countable.

In the formulation of the search problem, we will deal with two binary uncertain variables, corresponding to r.v.'s $X_{k,n}$ and $Y_{k,n}$. Hence, let us focus on a discrete uncertain variable X and its state space $\mathcal{X} = \{x_1, \ldots, x_N\}$. The possibility measure is a mapping $\Pi_X : 2^{\mathcal{X}} \to [0,1]$, where $2^{\mathcal{X}}$ is the set of all subsets of $\mathcal{X}$. Mapping Π_X satisfies three axioms: (1) $\Pi_X(\emptyset) = 0$; (2) $\Pi_X(\mathcal{X}) = 1$, and (3) the possibility of a union of disjoint events $A_1, A_2 \subseteq \mathcal{X}$ is given by $\Pi_X(A_1 \cup A_2) = \max[\Pi_X(A_1), \Pi_X(A_2)]$. Possibility measure Π_X is related to the possibility function π_X as follows:

$$\Pi_X(A) = \max_{x\in A} \pi_X(x)$$

for every $A \subseteq \mathcal{X}$. There is also a notion of *necessity* of an event $N_X(A)$, which is dual to $\Pi_X(A)$ in the sense that:

$$N_X(A) = 1 - \Pi_X(A^c), \tag{6}$$

where A^c is the complement of A in $\mathcal{X}$. One can interpret the necessity-possibility interval $[N_X(A), \Pi_X(A)]$ as the belief interval, specified by the lower and upper probabilities in the sense of Wiley [20]. Note that for a binary variable $X \in \{0,1\}$, this interval can be expressed for event $A = \{1\}$ as $Pr\{X = 1\} \in [N_X(1), \Pi_X(1)] = [1 - \Pi_X(0), \Pi_X(1)]$, where, due to normalisation, the following condition must be satisfied: $\max\{\Pi_X(0), \Pi_X(1)\} = 1$.

IV. Possibilistic search: theoretical formulation

A. Information state

Current knowledge about the count and positions of targets, in the framework of possibility theory, is represented by two posteriors: (1) the posterior possibility of target presence $\Pi_{k,n}^1 = \Pi_{X_{k,n}}(\{1\}|\mathbf{Z}_{1:k})$, and (2) the posterior possibility of target absence $\Pi_{k,n}^0 = \Pi_{X_{k,n}}(\{0\}|\mathbf{Z}_{1:k})$. We need both of them, because $\Pi_{k,n}^0$ cannot be worked out from $\Pi_{k,n}^1$ and vice verse. Consequently, during the search, *two posterior possibility maps* need to be updated sequentially over time, $\mathbf{\Pi}_k^1 = [\Pi_{k,n}^1]$ and $\mathbf{\Pi}_k^0 = [\Pi_{k,n}^0]$, where $n = 1, \ldots, n_c$.

Suppose now that the probability of detection is specified by an interval value, that is

$$D_{k,n} \in [\underline{D}_{k,n}, \overline{D}_{k,n}] \tag{7}$$

where $\underline{D}_{k,n}$ and $\overline{D}_{k,n}$ represent the lower and upper probability of this interval, respectively. Because detection event is a binary variable, due to reachability constraint for probability intervals [31], (7) implies that the probability of non-detection is in interval $[1 - \overline{D}_{k,n}, 1 - \underline{D}_{k,n}]$. Then, via normalisation we can express the possibility of detection $D^1_{k,n}$ and the possibility of non-detection $D^0_{k,n}$ (in cell n at time k) as[1]:

$$D^1_{k,n} = \frac{\overline{D}_{k,n}}{\max\{1 - \underline{D}_{k,n}, \overline{D}_{k,n}\}} \tag{8}$$

$$D^0_{k,n} = \frac{1 - \underline{D}_{k,n}}{\max\{1 - \underline{D}_{k,n}, \overline{D}_{k,n}\}}. \tag{9}$$

satisfying $\max\{D^0_{k,n}, D^1_{k,n}\} = 1$. Interval $[1 - D^0_{k,n}, D^1_{k,n}]$ represents the necessity-possibility interval for the probability of detection.

In general, the probability of detection $D_{k,n}$, as well as the two possibilities $D^0_{k,n}$ and $D^1_{k,n}$, are typically dependent on the distance $d_{n,k}$ between the nth grid cell and the searching agent position at time k.

In a similar manner we can also assume that the probability of false alarm is specified by an interval value, that is $F_{k,n} \in [1 - F^0_{k,n},\ F^1_{k,n}]$, where $F^0_{k,n}$ and $F^1_{k,n}$ represent the possibility of no false alarm and the possibility of false alarm (in cell n at time k), respectively.

Next we explain how to sequentially update, during the search, the two posterior possibility maps, $\mathbf{\Pi}^1_k$ (for target presence) and $\mathbf{\Pi}^0_k$ (for target absence). The proposed update equations follow from (2) and (3), after application of the Bayes' rule analog modifications, specific for the possibilistic framework, see [23], [33].

Given $\mathbf{\Pi}^1_{k-1}$ and detection set $\mathbf{Z}_k$, if none of the dectections in $\mathbf{Z}_k$ falls into the nth cell, the possibility of target presence in the nth cell is updated as follows:

$$\Pi^1_{k,n} = \frac{D^0_{k,n}\,\Pi^1_{k-1,n}}{\max\{D^0_{k,n}\,\Pi^1_{k-1,n},\ F^0_{k,n}\,\Pi^0_{k-1,n}\}}. \tag{10}$$

for $n = 1, \dots, n_c$. Similarly, in this case $\Pi^0_{k,n}$ is updated according to:

$$\Pi^0_{k,n} = \frac{F^0_{k,n}\,\Pi^0_{k-1,n}}{\max\{D^0_{k,n}\,\Pi^1_{k-1,n},\ F^0_{k,n}\,\Pi^0_{k-1,n}\}}. \tag{11}$$

If a detection from $\mathbf{Z}_k$ falls into the nth cell, then the update equation for $\Pi^1_{k,n}$ can be expressed as:

$$\Pi^1_{k,n} = \frac{D^1_{k,n}\,\Pi^1_{k-1,n}}{\max\{D^1_{k,n}\,\Pi^1_{k-1,n},\ F^1_{k,n}\,\Pi^0_{k-1,n}\}}. \tag{12}$$

[1]Another method for the specification of a possibility function from a probability mass function expressed by probability intervals would be via the maximum specificity criterion [32].

And finally, in this case the update equation for $\Pi^0_{k,n}$ is given by:

$$\Pi^0_{k,n} = \frac{F^1_{k,n}\,\Pi^0_{k-1,n}}{\max\{D^1_{k,n}\,\Pi^1_{k-1,n},\ F^1_{k,n}\,\Pi^0_{k-1,n}\}} \tag{13}$$

Note that the *probability* of target presence in each cell of the search area, using the described possibilistic approach, is expressed by an interval value, i.e.

$$P_{k,n} \in [1 - \Pi^0_{k,n},\ \Pi^1_{k,n}] \tag{14}$$

for $n = 1, \dots, n_c$, where $\max\{\Pi^0_{k,n},\ \Pi^1_{k,n}\} = 1$. Initially, at time $k = 0$ (before any sensing action), the posterior possibility maps are set to:

$$\Pi^0_{0,n} = \Pi^1_{0,n} = 1, \tag{15}$$

meaning that $P_{0,n} \in [0, 1]$, for $n = 1, \dots, n_c$. This is an expression of total ignorance about the probability of target presence.

B. Epistemic Reward

Let us first define the amount of information contained in the information state, represented by two posterior possibility maps, $\mathbf{\Pi}^1_k$ and $\mathbf{\Pi}^0_k$. Various information (i.e. uncertainty) measures in the context of non-additive probabilistic frameworks have been proposed in the past [34]–[36]. We adopt the principle that epistemic uncertainty, on the continuous state space, corresponds to the volume under the possibility function [25], [36]. Applying this principle to our problem results in the following definition of *possibilistic entropy* $\mathcal{G}_k$ contained in posterior maps $\mathbf{\Pi}^1_k$ and $\mathbf{\Pi}^0_k$:

$$\mathcal{G}_k = 1 - \frac{1}{n_c}\sum_{n=1}^{n_c} |\Pi^0_{k,n} - \Pi^1_{k,n}|. \tag{16}$$

Note that at $k = 0$, when $\Pi^0_{0,n} = \Pi^1_{0,n} = 1$, we have $\mathcal{G}_0 = 1$. This means that initially (at the start of the search), the amount of information contained in the initial information state, is zero (representing total ignorance). As the searching agent moves and collects measurements, it gains knowledge and as a result either $\Pi^0_{k,n}$ or $\Pi^1_{k,n}$ will reduce its value in some cells (keeping in mind that $\max\{\Pi^0_{k,n},\ \Pi^1_{k,n}\} = 1$), thus reducing the possibilsitic entropy $\mathcal{G}_k$.

The reward function for search in the framework of the possibility theory, is defined as the *reduction of possibilistic entropy* of the information state expressed by maps $\mathbf{\Pi}^1_k$ and $\mathbf{\Pi}^0_k$. Mathematically this is expressed as:

$$\mathcal{R}_k(\alpha) = \mathcal{G}_{k-1} - \mathcal{E}\{\mathcal{G}_k(\alpha)\} \tag{17}$$

where as before $\alpha \in \mathcal{A}_k$ is an action from the set of admissible actions at time k and $\mathcal{E}$ is the expectation with respect to the (random) measurement set $\mathbf{Z}_k(\alpha)$. In order to simplify the computation, we will consider only a single realisation for $\mathbf{Z}_k(\alpha)$: the one which results in detection(s) at those cells characterised by $\Pi^1_{k,n} - \Pi^0_{k,n} > \zeta$, where ζ is a threshold close to 1. Finally, the searching agent chooses the action which maximises the reward, as in (5).

The search mission is terminated when the reduction of *possibilsitic entropy* falls below a specified threshold, i.e. when $\mathcal{G}_{k-1} - \mathcal{G}_k < \xi$.

V. Numerical results

Consider a rectangular search area $\mathcal{S}$ of size 100 km $\times$ 90 km, discretised into $n_c = 100 \times 90 = 9000$ resolution cells, each covering 1 km^2. A total of 80 targets are randomly placed in two corners of the search area (northwest and south-east). A typical scenario is shown in Fig. 1, where black coloured squares indicate target locations.

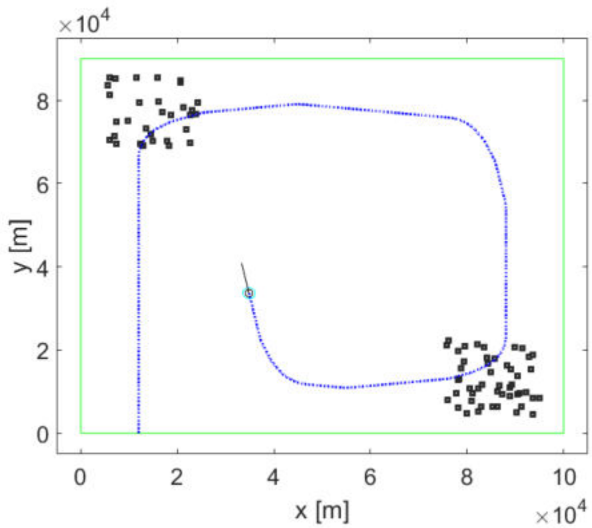

Fig. 1. Simulation setup: the small black squares indicate the true target locations; the blue dotted line is the trajectory of the searching agent up to $k = 150$ steps.

The probability of detection D is modelled as a (monotonically decreasing) function of the distance between the nth grid cell and the searching agent position at time k. For illustration, the following mathematical model was used[2]:

$$D(d; \mu, \sigma) = 1 - \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \qquad (18)$$

[2]Note that any other model can be adopted.

where $d \geq 0$ is the distance, while $\mu > 0$ and $\sigma > 0$ are modeling parameters. Fig. 2 displays the imprecise model of the probability of detection D as a function of distance d, using (18) with two sets of parameters μ and σ (the green-coloured area). The search algorithm described in Sec. IV, is using this imprecise model for its search mission. The model provides the information for $D^1_{k,n}$ and $D^0_{k,n}$, introduced in (8) and (9), respectively. The true value of the probability of detection, which is used in simulated measurement generation (but which is unknown to the search algorithm) is plotted with the solid red line in Fig. 2. This curve is also based on model (18), using one particular pair of μ and σ values.

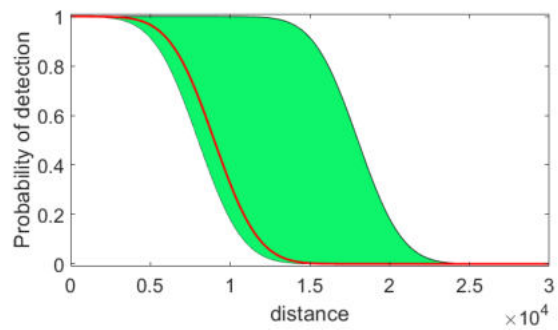

Fig. 2. The green area represents the imprecise model of the probability of detection D as a function of distance (range) to the target. The true D curve is plotted using the red solid line.

With this specification of D, the probability of detecting a target more than a certain distance (which depends on parameters μ and σ) is practically zero. Assuming 360° coverage, the sensing area $\mathcal{L}_k$ is a circular area. The spatial distribution of false alarms is assumed to be uniform over $\mathcal{L}_k$, with a false-alarm probability is $F_{k,n} = 0.005$. For simplicity, we will assume that this parameter is known as the precise value to the search algorithm of Sec. IV. The threshold parameter ζ is set to 0.8.

Sensor measurements are affected by additive Gaussian noise with the standard deviation in range and azimuth of 100m and 1°, respectively. An additional assumption is that there is at most one target per cell and one detection per cell.

Searching agent (UAV) motion is modelled by the coordinated turn (CT) model [37] with the turning rate taking values from the set

$$\Omega = \{-0.4, -0.3, -0.2, -1, 0, 0.1, 0.2, 0.3, 0.4\}$$

(the units are °/s). We consider one-step ahead path planning, with action space $\mathcal{A}_k$ defined as a Cartesian product $\mathcal{A}_k = \Omega \times \Delta$. Here Δ is the set of time intervals of CT motion (with the selected turning rate), adopted as $\Delta = 60s, 120s$.

Let us now compare the average search performance of the proposed possibilistic search with the probabilistic search. The adopted metric for search performance is the Optimal Sub-pattern Assignments (OSPA) error, because it expresses in a mathematically rigorous manner the error both in the target position estimate and in the target number (cardinality error) [38]. The parameters of OSPA error used are: cut-off $c = 50$km and order $p = 1$. Mean OSPA error is estimated by averaging over 100 Monte Carlo runs, with a random placement of targets on every run (but always in the same two regions of Fig. 1). Because the search duration is random, for the sake of averaging the OSPA error, we fixed the duration of search mission to $k = 220$ time steps.

In order to apply the (conventional) probabilistic search to the scenario described above, we must adopt a precise (rather than an interval-valued) probability of detection. We will consider two cases: (a) when the true probability of detection versus range (i.e. the red line in Fig. 2) is known; this case is referred to as the *matched-case*; (b) given the interval-valued probability of detection (green area in Fig. 2), we choose the mid-point of the interval at a given range, as the true value; this case is referred to as the *model-mismatched* case. Case (a) is ideal and is expected to result in the best performance, whereas case (b), because it uses an incorrect value of the probability of detection, is expected to perform worse.

The resulting three mean OSPA errors are presented in Fig. 3: black line for possibilistic search; blue line for probabilistic - matched case; red line for probabilistic - mismatched case. Initially (up to $k = 38$), the three curves are identical, because not a single target has been detected up to that time. After that we see a dramatic difference. Of the three comparing methods, as expected, the best performance (i.e. the smallest OSPA error) is achieved using the probabilistic - matched case (ideal case). The possibilistic solution, which operates using the available interval-valued probability of detection, is fairly close to the ideal case. This is remarkable, because the interval (the green area in Fig. 2) is fairly broad. Finally, the probabilistic - mismatched solution performs poorly.

VI. Conclusions

The paper presented a novel autonomous search algorithm for localisation of targets. The key novelty is that the solution is formulated in the framework of the possibility theory, instead of the traditional probability theory. The main rational for the possibilistic formulation is its robustness in situations where the probabilistic models are partially known. In this paper, we have demonstrated this feature by considering interval-valued probability of detection (as a function of range). The numerical results show that the (conventional) probabilistic solution performs sightly better when the correct precise model of probability of detection is known (the model-match case), but notably worse if an incorrect precise model is adopted (model-mismatched case).

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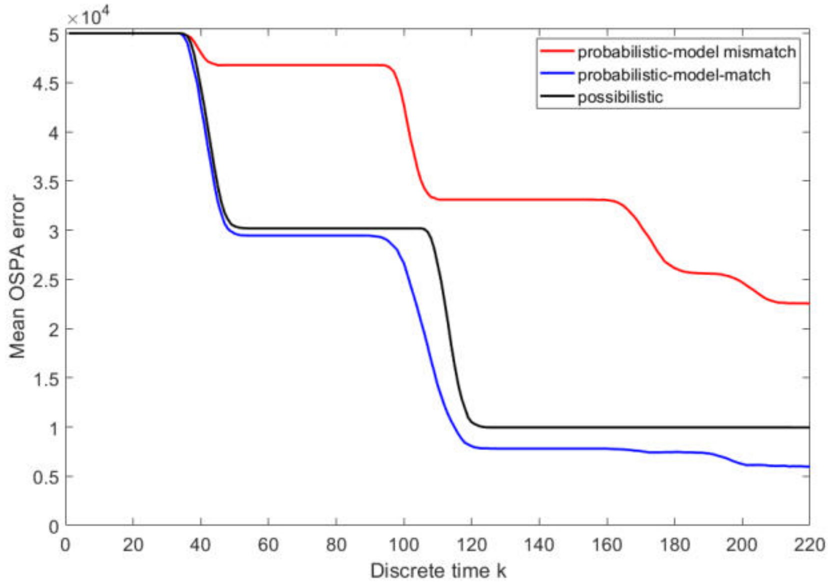

Fig. 3. Mean OSPA errors obtained from 100 Monte Carlo runs.

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